Problem Set 3

Problem 1 *Suppose that* $Y = X\beta + e$ *where* $e \sim N(0_{T\times 1}, \Omega)$ *and* $\Omega \neq \sigma^2 I_T$.

(1) Show that the OLS estimator is unbiased.

(2) Can we still use the t-statistics by simply assuming homoscedasticity? Why or why not?

(3) Derive the GLS estimator when Ω *is known. Is the GLS estimator BLUE? Why or why not?*

(4) Show that the OLS estimator is not BLUE.

(5) If the GLS estimator is not feasible, what would be an alternative for doing t-test?

Problem 2 *Consider the following regression equation*

$$Y = X_1\beta_1 + X_2\beta_2 + X_3\beta_3 + e, \quad e \sim N(0_{T\times 1}, \sigma^2 I_T)$$

Suppose that $X_2 = 3X_1 - 2X_3$.

(1) Are the classical assumptions satisfied? If not, explain why.

(2) Are all the coefficients identified? If not, explain why and suggest a remedy.

Problem 3 *Consider the following regression equation*

$$\begin{aligned} \textit{DGP:} \quad & Y = X_1\beta_1 + e, \quad e \sim N(0_{T\times 1}, \sigma^2 I_T) \\ \textit{Reg. equation:} \quad & Y = X_1\beta_1 + X_2\beta_2 + e, \quad e \sim N(0_{T\times 1}, \sigma^2 I_T) \end{aligned}$$

Is $\hat{\beta}_1$ *unbiased? How does adding irrelevant variable* X_2 *affect the distribution of* $\hat{\beta}_1$ *in comparison with the case that* X_2 *is not included? Prove your answer.*

Problem 4 *Consider the following regression equation*

$$\begin{aligned} \textit{DGP:} \quad & Y = X_1\beta_1 + X_2\beta_2 + e, \quad e \sim N(0_{T\times 1}, \sigma^2 I_T) \\ \textit{Reg. equation:} \quad & Y = X_1\beta_1 + e, \ e \sim N(0_{T\times 1}, \sigma^2 I_T) \end{aligned}$$

Is $\hat{\beta}_1$ *unbiased? Prove your answer.*

Problem 5 *Sang-Hee wants to forecast the dependent variable at time* $T+1$, y_{T+1} *by estimating the following regression equation*

$$Y = X\beta + e, \quad e \sim N(0_{T\times 1}, \sigma^2 I_T)$$

(1) What is the forecasts of y_{T+1}*?*

(2) What is the nature of prediction error? Explain your answer by deriving the variance of prediction error.

Problem 6 *Briefly explain the following notions:*

(1) Unit root process
(2) Unit root test
(3) Cointegration
(4) Stationary process
(5) Spurious regression

Problem 7 *Consider the following model*

$$y_i = \mu + e_i, \quad e_i \sim \text{i.i.d.} N(0, 1)$$

(1) Construct the log likelihood function.
(2) Derive the maximum likelihood estimator of μ.
(3) Derive Var($\hat{\mu}$).